TRIGONOMETRY VOCABULARY

Angle: the figure formed by two rays sharing a common endpoint or vertex.

Positive Angle – an angle measured in with a counterclockwise rotation

Negative Angle- an angle measured in with a clockwise rotation

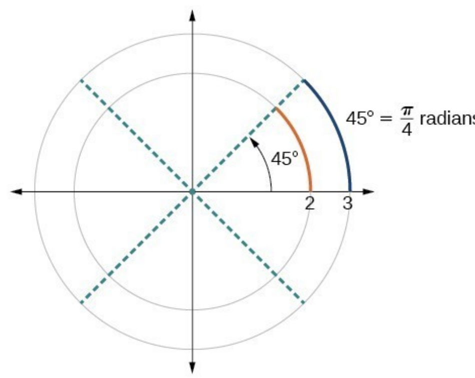
Measure- the rotation about the vertex required to move from one ray to another.

Degree- a measurement of an angle. A circle is broken up into 360°

Radian- the ratio of the length of an arc to the length of its related radius. A circle is broken up into 2π radians.

DEGREES AND RADIANS

The two most commonly used measures for angles are degrees and radians. There are 360 degrees in a full circle (a right angle is 90 degrees), and 2π radians in a full circle (there are $\pi/2$ radians in a right angle). If you calculate it out, which is not necessary, there are about 57 degrees in a radian.



Students typically learn about degrees before they learn about radians, which brings up the question: Why learn about radians if degrees are good enough for measuring angles? There are two reasons, but both are grounded in the fact that the radian is a "*unit-less*" measure.

The radian is defined to be the ratio of the length of the arc of a circle to the length of the radius of the circle, where **each length is measured in the same unit**. Therefore, when you divide to get the radian measure of the angle, the units for the two lengths cancel, and you end up with a measure that has no units! This is cool because radians can be used mathematically to solve problem without having to apply a conversion factor. Very very cool.

Finally, the number of degrees in a circle were originally chosen because they were easy to mathematically work with. On the other hand, there is something aesthetically pleasing about the definition of radian measure, as it is a simple ratio. It also makes important mathematic formulas simpler and easier to work with. For example, Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$ would have to be written in a considerably uglier form with degrees rather than radians.

CONVERTING BETWEEN DEGREES AND RADIANS

If a circle of radius 1 is drawn with the vertex of an angle at its center, then the measure of this angle in radians (rad) is the length of the arc that subtends the angle. In short, we can say that 360 degrees is equivalent to the ratio of 2π radians. This gives us a conversion factor that we can use to switch between radians and degrees. To convert from degrees to radians we can simply multiply by the ratio of (2π/360), or in reduced form (π/180), since the two quantities are equivalent.

1. To convert degrees to radians, multiply by $\frac{\pi}{180}$
2. To convert radians to degrees, multiply by $\frac{180}{\pi}$

Converting Between Radians and degrees:

Example 1) $\dfrac{7\pi}{6}$

$$\frac{7\pi}{6}\left(\frac{180°}{\pi}\right) = 7 \times 30° = 210°$$

Example 2) 225°

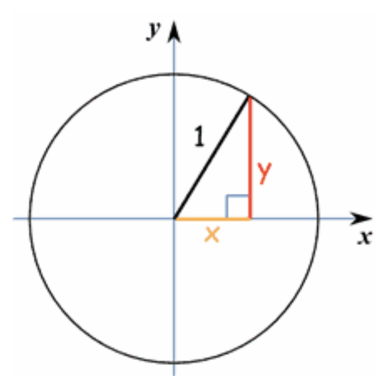
$$225°\left(\frac{\pi}{180°}\right) = \frac{225°}{180°}\pi = \frac{5}{4}\pi \text{ or } \frac{5\pi}{4}$$

Example 3) $\dfrac{\pi}{3}$

$$\frac{\pi}{3}\left(\frac{180°}{\pi}\right) = \frac{180°}{3} = 60°$$

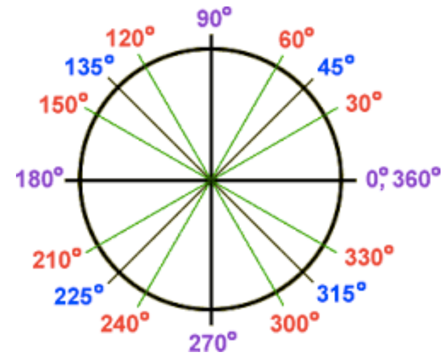
CIRCLES AND TRIANGLES AND TRIGONOMENTRY

Just like the name suggests, trigonometry is the study of the sides and angles in a triangle. Curiously enough, the both the properties of circles and the properties of triangles (specifically, right triangles) converge in trigonometry.



The unit circle is a platform for describing all the possible angle measures from 0 to 360 degrees, all the negatives of those angles, plus all the multiples of the positive and negative angles from negative infinity to positive infinity. In other words, the unit circle shows you all the angles that exist.

POSITIVE AND NEGATIVE ANGLES



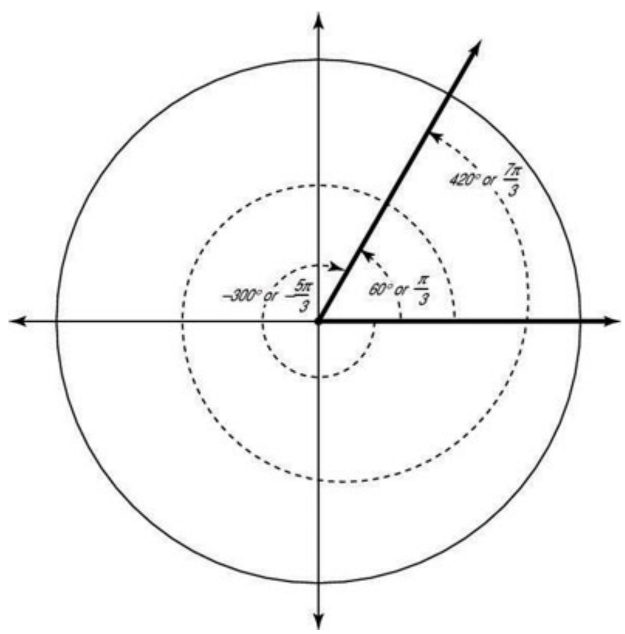
The positive angles on the unit circle are measured with the initial side on the positive x-axis and the terminal side moving counterclockwise around the origin. These angles are positive and can be measured in degrees or radians.

Notice that the terminal sides of the angles measuring 30 degrees and 210 degrees, 60 degrees and 240 degrees, and so on form straight lines. This fact is to be expected because the angles are 180 degrees apart, and a straight angle measures 180 degrees. You see the significance of this fact when you deal with the trig functions for these angles.

NEGATIVE ANGLES

Just when you thought that angles measuring up to 360 degrees or 2π radians was enough for anyone, you're confronted with the reality that many of the basic angles have negative values and even multiples of themselves. If you measure angles clockwise instead of counterclockwise, then the angles have negative measures:

A 30-degree angle is the same as an angle measuring –330 degrees, because they have the same terminal side. Likewise, an angle of –30 degrees is the same as an angle measuring 330 degrees.



But wait — you have even more ways to name an angle. By doing a complete rotation of two (or more) and adding or subtracting 360 degrees or a multiple of it before settling on the angle's terminal side, you can get an infinite number of angle measures, both positive and negative, for the same basic angle.

For example, an angle of 60 degrees has the same terminal side as that of a 420-degree angle and a –300-degree angle. The figure shows many names for the same 60-degree angle in both degrees and radians.

Although this name-calling of angles may seem pointless at first, there's more to it than arbitrarily using negatives or multiples of angles just to be difficult. The angles that are related to one another have trig functions that are also related, if not the same.