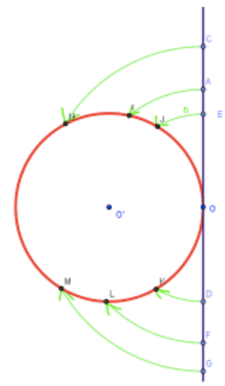
Trigonometric Functions on the Unit Circle

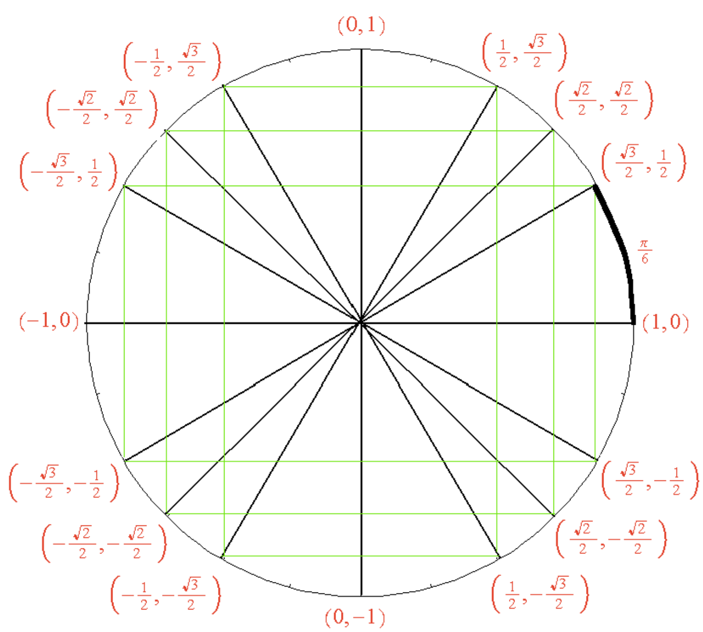
Many years ago, some genius figured out that because the sine and cosine functions were wave-like, we could essentially wrap those functions around a circle. This process creates what is literally called a "wrapping function." The benefit of a wrapping function is that we can see the whole cycle of the function on a compact graph. By understanding the pattern, in theory we will are to understand the complete function at any point.



The **Unit Circle is simply a circle of radius 1 centered at the origin**. The radius is intentionally set at one so the arc length of any angle corresponds exactly to the angle's radian measure. Setting up a circle this way is similar to scoring a test on a 100 point scale regardless of the number of questions on the test. Both are useful for extending to any test or function. The Unit Circle simply provides us with a way of mapping a real number input value for a trigonometric function to a real number output value. Let's take a look at an example.

The 16-Point Unit Circle

When we wrap the sine and cosine function around the Unit Circle the x values will correspond to the cosine and the y values will correspond to the sine function. This is in part due to the fact that the ratios expressed by the sine and cosine functions are directly related. For example, if the hypotenuse is fixed, as we increase an angle the adjacent side will get smaller as the opposite side gets larger. You can visually see this on the diagram to the right.



The 16-Point Unit Circle is a Unit Circle with some common angles already specified. These angles are 0, 30, 45, 60 and 90 degrees in quadrant 1 and multiples of those angles as we go around the circle. You will want to refer to this circle frequently during unit sections with lots of trigonometry functions. If you are able, it is best to memorize these exact function values so you can quickly perform calculations involving them.

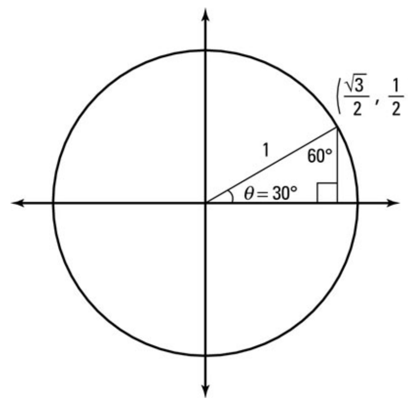
Using the 16-Point Unit Circle

It is relatively easy to read sine and cosine values off the Unit Circle. The basic idea is that the sine and cosine relate directly to the x and y values of their corresponding location on the Unit Circle. The values of the cosine and sine will vary from negative 1 to positive 1 as you progress around the Unit Circle.

Example 1

Find the cosine, sine and tangent of 30 degrees using the Unit Circle.

Answer: Using the Unit Circle, we find that a 30 degree angle intersects our unit circle at the point $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. This give us a cosine of $\frac{\sqrt{3}}{2}$ and a sine of $\frac{1}{2}$. The tangent of 30 degrees will be $\frac{\sqrt{3}}{2} / \frac{1}{2}$. This reduces to $\sqrt{3}$. Looking at the graph, we also see that the cosine and sine of 60 degrees will be the reverse of those values, sine is $\frac{\sqrt{3}}{2}$ and the cosine is $\frac{1}{2}$.



Practice

Using the Unit Circle, find the cosine, sine and tangent of these common angles.

Degrees	Radians	Sine	Cosine	Tangent
0°				
30°				
45°				
60°				
90°				
120°				
135°				
150°				
180°				